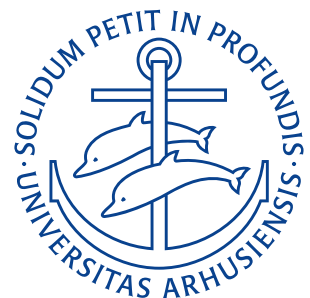


Department of Mechanical and Production Engineering
Aarhus University

Formula Sheet

Author: Noah Rahbek Bigum Hansen
Student ID: 202405538
Course: Electromagnetism & Electronics — 290231U007
Date: February 25, 2026



Contents

1	Constant Definitions	3
1.1	Coulomb Constant	3
2	Lecture 1: Electrostatics	3
2.1	Coulomb's Law	3
2.1.1	Principle of Superposition	3
2.2	Quantization of Charge	3
2.3	Electric Field at a point	4
2.3.1	Electric Field Produced by Point Charge	4
2.3.2	Electric Field due to a Dipole along dipole axis	4
2.3.3	Electric Field due to a Dipole along Dipole axis at Long Distance	4
2.3.4	Electric Field of a Charged Ring at Large Distance	5
2.3.5	Electric Field of a Charged Disk	5
2.3.6	Electric field from an infinite sheet	5
2.3.7	Electric field of a Conducting Sphere	6
2.4	Electric Field of an Insulating Sphere	6
2.5	Gauss' Law	7
2.5.1	Net electric flux through a surface caused by an electric field	7
2.6	Electric Potential Energy	7
2.6.1	Electrostatic Potential	8
2.6.2	Electrostatic Potential of a Plate Capacitor	8
3	Lecture 2: Fundamentals of Electric Circuits	8
3.1	Current, Voltage, and Resistance	8
3.1.1	Electric Current	8
3.1.2	Electric Voltage	8
3.1.3	Electric Resistance	9
3.1.4	Current Density	9
3.1.5	Resistivity	9
3.1.6	Conductivity	9
3.1.7	Resistance From Resistivity	9
3.1.8	Ohm's Law	10
3.1.9	Electric Power	10
3.2	Kirchoff's Circuit Laws	10
3.2.1	Kirchoff's Current Law (KCL)	10
3.2.2	Kirchoff's Voltage Law (KVL)	10
3.2.3	Resistors in Series	10
3.2.4	Voltage Divider Rule	11
3.2.5	Resistors in Parallel	11
3.2.6	Current Divider Rule	11
4	Lecture 3: Principles of Circuit Analysis	11
5	Lecture 4: Capacitors & Inductors	11
5.1	Capacitors	11
5.1.1	Capacitance	11
5.1.2	Capacitance of a Parallel-Plate Capacitor	11
5.1.3	I-V Relation for an Ideal Capacitor	12

5.1.4	Capacitance of a Cylindrical Capacitor	12
5.1.5	Capacitance of a Cylindrical Capacitor	12
5.1.6	Parallel Capacitance	12
5.1.7	Capacitors in Series	13
5.1.8	Potential Energy in a Capacitor	13
5.2	Inductors	13
5.2.1	Inductance	13
5.2.2	I-V Relation for an Ideal Inductor	13
5.2.3	Magnetic Field of an Inductor	13
5.2.4	Inductors in Series	14
5.2.5	Parallel Inductors	14
5.2.6	Energy Stored in an Inductor	14
5.2.7	Energy Density in an Inductor	14
5.3	RC and RL Circuits in the DC Regime	14
5.3.1	Charge in a (Charging) Capacitor as a Function of Time	14
5.3.2	Current in a (Charging) Capacitor as a Function of Time	15
5.3.3	Capacitive Time Constant	15
5.3.4	Inductive Time Constant	15
5.3.5	Current in an Inductor	15
6	Lecture 5: AC Circuits	16
6.1	AC Circuits	16
6.1.1	Periodic Signals	16
6.1.2	Average Value of a Signal Waveform	16
6.1.3	RMS value	16
6.2	Impedance	16
6.2.1	Impedance of a Resistor	16
6.2.2	Impedance of an Inductor	17
6.2.3	Impedance of a Capacitor	17
6.2.4	Total Impedance	17
6.2.5	Admittance	17
6.3	Transient Analysis	18
6.3.1	1st Order Differential Equation for RC Circuits	18
6.3.2	2nd Order Differential Equation for RLC Circuits	18
7	Lecture 6: Frequency Analysis	18
7.1	Frequency Response Function	18
7.1.1	Phase Shift due to the Frequency Response Function	18
7.2	Passive Filters	19
7.2.1	Cutoff Frequency	19
7.2.2	Low-Pass Filter	19
7.2.3	High-Pass Filter	20
7.2.4	Bandpass Filter	20
8	Lecture 7: Diodes	22
8.1	Diodes	22
8.1.1	Net Diode Current in forward bias mode	22

1 Constant Definitions

1.1 Coulomb Constant

$$k = \frac{1}{4\pi\epsilon_0} = 8,99 \cdot 10^9 \text{ Nm}^2/\text{C}^2.$$

Symbols

k	Coulomb constant	$8,99 \cdot 10^9 \text{ Nm}^2/\text{C}^2$
ϵ_0	Permittivity constant	$8,85 \cdot 10^{-12} \text{ C}^2/\text{Nm}^2$

Lecture 1: Electrostatics

26. Januar 2026

2.1 Coulomb's Law

$$\mathbf{F} = k \frac{q_1 q_2}{r^2} \hat{r}.$$

Assumptions

- Point charges

Symbols

$\mathbf{F}$	Electrostatic force	N
k	Coulomb constant	$8,99 \cdot 10^9 \text{ Nm}^2/\text{C}^2$
q_1, q_2	Charges	C
r	Distance	m
$\hat{r}$	Unit vector	—

2.1.1 Principle of Superposition

$$\mathbf{F}_{1,\text{net}} = \sum_{i=1}^n \mathbf{F}_{1i}.$$

Symbols

$\mathbf{F}_{1,\text{net}}$	Net force	N
$\mathbf{F}_{1i}$	Force contribution on particle 1 from particle i	N

2.2 Quantization of Charge

$$q = \pm n \cdot e.$$

Symbols

q	Charge	C
n	Integer number ($n \in \mathbb{Z}$)	—
e	Elementary charge	$1,6 \cdot 10^{-19} \text{ C}$

2.3 Electric Field at a point

$$\mathbf{E}(x, y, z) = \frac{\mathbf{F}(x, y, z)}{q_0}.$$

Symbols

$\mathbf{E}(x, y, z)$	Electric field at point (x, y, z)	$\frac{\text{N}}{\text{C}}$
$\mathbf{F}(x, y, z)$	Force on test particle at point (x, y, z)	N
q_0	Test charge	C

2.3.1 Electric Field Produced by Point Charge

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2}.$$

Symbols

E	Electric field strength	$\frac{\text{N}}{\text{C}}$
ϵ_0	Permittivity constant	$8,85 \cdot 10^{-12} \text{ C}^2/\text{Nm}^2$
q	Charge	C
r	Distance	m
$\hat{r}$	Unit vector	—

2.3.2 Electric Field due to a Dipole along dipole axis

$$E(z) = E_{(+)}(z) + E_{(-)}(z) = \frac{q}{4\pi\epsilon_0 z^2} \left(\frac{1}{1 - \frac{d^2}{2z}} - \frac{1}{1 + \frac{d^2}{2z}} \right) = \frac{q}{2\pi\epsilon_0 z^3} \left(\frac{d}{\left(1 - \left(\frac{d}{2z}\right)^2\right)^2} \right).$$

Symbols

E	Electric field strength	$\frac{\text{N}}{\text{C}}$
$E_{(+)}, E_{(-)}$	Contribution to E from the positive and negative part of the dipole	$\frac{\text{N}}{\text{C}}$
ϵ_0	Permittivity constant	$8,85 \cdot 10^{-12} \text{ C}^2/\text{Nm}^2$
z	Distance along axis	m
d	Separation distance	m
q	Charge	C

2.3.3 Electric Field due to a Dipole along Dipole axis at Long Distance

$$E = \frac{1}{2\pi\epsilon_0} \frac{qd}{z^3} = \frac{1}{2\pi\epsilon_0} \frac{p}{z^3}.$$

Assumptions

- $z \gg d$, i.e. the distance between charges is much smaller than the distance from the charges you are measuring ("normal" case)

Symbols

E	Electric field strength	$\frac{\text{N}}{\text{C}}$
ϵ_0	Permittivity constant	$8,85 \cdot 10^{-12} \text{ C}^2/\text{Nm}^2$
z	Distance along dipole axis	m
d	Separation distance	m
q	Charge	C
p	Electric dipole moment ($p = qd$)	Cm

2.3.4 Electric Field of a Charged Ring at Large Distance

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{z^2}.$$

Assumptions

- $z \gg R$, i.e. distance from the ring is much greater than the radius of the ring

Symbols

E	Electric field strength	$\frac{\text{N}}{\text{C}}$
ϵ_0	Permittivity constant	$8,85 \cdot 10^{-12} \text{ C}^2/\text{Nm}^2$
q	Size of charge in the ring	C
z	Distance perpendicular to the radius of the ring	m

2.3.5 Electric Field of a Charged Disk

$$E = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right).$$

Assumptions

- Infinitesimally thin disk

Symbols

E	Electric field strength	$\frac{\text{N}}{\text{C}}$
ϵ_0	Permittivity constant	$8,85 \cdot 10^{-12} \text{ C}^2/\text{Nm}^2$
σ	Surface charge density	C/m^2
z	Distance perpendicular to the disk	m
R	radius of disk	m

2.3.6 Electric field from an infinite sheet

$$E = \frac{\sigma}{2\epsilon_0}.$$

Assumptions

- Infinitesimally thin infinite sheet

Symbols

E	Electric field strength	$\frac{\text{N}}{\text{C}}$
σ	Surface charge density	C/m^2
ϵ_0	Permittivity constant	$8,85 \cdot 10^{-12} \text{ C}^2/\text{Nm}^2$

2.3.7 Electric field of a Conducting Sphere

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} = k \frac{q}{r^2}.$$

Assumptions

- Positive electric charge Q distributed uniformly throughout the volume of a conducting sphere.
- Spherical symmetry
- No charge enclosed in conductor

Symbols

E	Electric field strength	$\frac{\text{N}}{\text{C}}$
ϵ_0	Permittivity constant	$8,85 \cdot 10^{-12} \text{ C}^2/\text{Nm}^2$
q	Charge	C
r	Distance	m
k	Coulomb constant	$8,99 \cdot 10^9 \text{ Nm}^2/\text{C}^2$

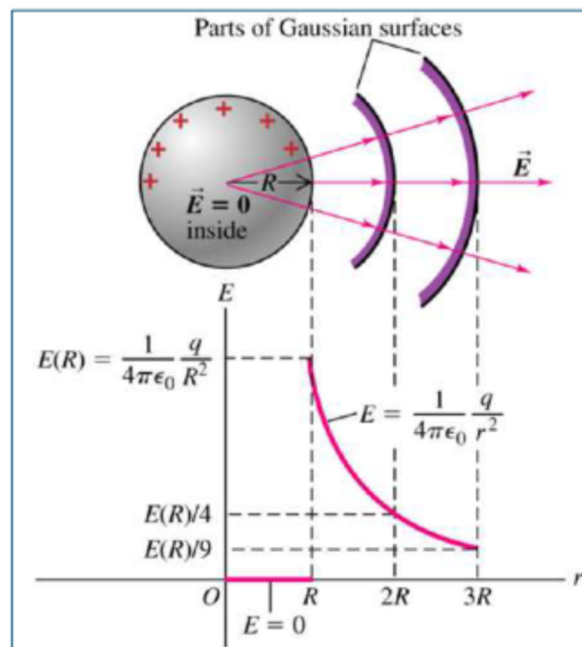


Figure 1: E-field propagation from a conducting sphere.

2.4 Electric Field of an Insulating Sphere

Outside :	$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$
Inside :	$E = \frac{q}{4\pi\epsilon_0 R^3} r.$

Assumptions

- Positive electric charge Q distributed uniformly throughout the volume of an insulating sphere.
- Spherical symmetry

Symbols

E	Electric field strength	$\frac{\text{N}}{\text{C}}$
r	Distance	m
R	Radius	m
ϵ_0	Permittivity constant	$8,85 \cdot 10^{-12} \text{ C}^2/\text{Nm}^2$
q	Charge	C

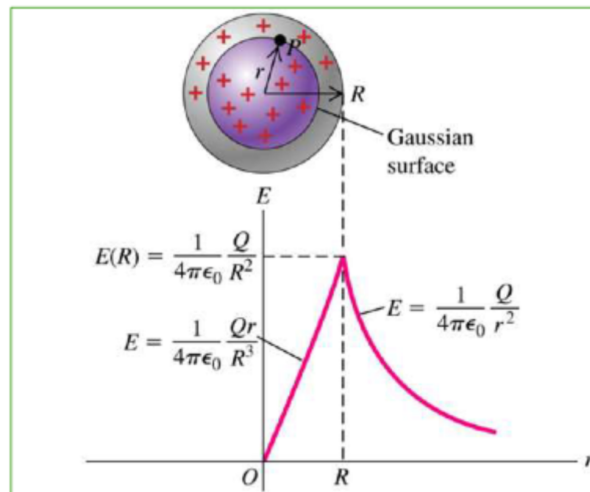


Figure 2: E-field propagation through an insulating sphere.

2.5 Gauss' Law

$$\Phi = \frac{q_{\text{enc}}}{\epsilon_0}.$$

Assumptions

- Vacuum (air)

Symbols

Φ	Electric flux	Vm
q_{enc}	Enclosed charge	C
ϵ_0	Permittivity constant	$8,85 \cdot 10^{-12} \text{ C}^2/\text{Nm}^2$

2.5.1 Net electric flux through a surface caused by an electric field

$$\Phi = \oint E \cdot dA.$$

Symbols

Φ	Electric flux	Vm
E	Electric field vector	$\frac{\text{N}}{\text{C}}$
A	Area vector	m^2

2.6 Electric Potential Energy

$$U_E = q_0 E \Delta d.$$

Symbols

U_E	Electric potential energy	J
q_0	Test charge	C
E	Electric field strength	$\frac{\text{N}}{\text{C}}$
d	Separation distance	m

2.6.1 Electrostatic Potential

$$V_E = \frac{U_E}{q_0}.$$

Symbols

V_E	Electrostatic potential	V
U_E	Electric potential energy	J
q_0	Test charge	C

2.6.2 Electrostatic Potential of a Plate Capacitor

$$V_E = Ed.$$

Symbols

V_E	Electrostatic potential	V
E	Electric field strength	$\frac{\text{N}}{\text{C}}$
d	Separation distance	m

Lecture 2: Fundamentals of Electric Circuits

28. Januar 2026

3.1 Current, Voltage, and Resistance**3.1.1 Electric Current**

$$I = \frac{dq}{dt}.$$

Symbols

I	Current	A
q	Charge	C
t	Time	s

3.1.2 Electric Voltage

$$V = \frac{dW}{dQ}.$$

Symbols

V	Voltage	V
W	Work	J
Q	Total charge	C

3.1.3 Electric Resistance

$$R = \frac{V}{I}.$$

Symbols

R	Resistance	Ω
V	Voltage	V
I	Current	A

3.1.4 Current Density

$$J = \frac{I}{A}.$$

Symbols

J	Current density	A/m ²
I	Current	A
A	Cross-sectional area	m ²

3.1.5 Resistivity

$$\rho = \frac{E}{J}.$$

Symbols

ρ	Resistivity	$\Omega \text{ m}$
E	Electric field strength	$\frac{\text{N}}{\text{C}}$
J	Current density	A/m ²

3.1.6 Conductivity

$$\sigma = \frac{1}{\rho}.$$

Symbols

σ	Conductivity	$\Omega^{-1} \text{ m}^{-1}$
ρ	Resistivity	$\Omega \text{ m}$

3.1.7 Resistance From Resistivity

$$R = \frac{\rho L}{A}.$$

Symbols

R	Resistance	Ω
ρ	Resistivity	$\Omega \text{ m}$
L	Length	m
A	Cross-sectional area	m ²

3.1.8 Ohm's Law

$$I \propto V.$$

Symbols

I	Current	A
V	Voltage	V

3.1.9 Electric Power

$$P = VI = I^2 R = \frac{V^2}{R}.$$

Symbols

P	Electrical power	W
V	Voltage	V
I	Current	A
R	Resistance	Ω

3.2 Kirchoff's Circuit Laws**3.2.1 Kirchoff's Current Law (KCL)**

$$\sum_{n=1}^N I_n = 0.$$

Symbols

I_n	Current at node n	A
N	Amount of nodes	—

3.2.2 Kirchoff's Voltage Law (KVL)

$$\sum_{n=1}^N V_n = 0.$$

Symbols

V_n	Voltage difference between two points	V
N	Amount of points	N

3.2.3 Resistors in Series

$$R_{\text{EQ}} = \sum_{n=1}^N R_n.$$

Symbols

R_{eq}	Equivalent resistance	Ω
R_n	Resistance of resistor n	Ω
N	Amount of resistors in series	—

3.2.4 Voltage Divider Rule

$$V_n = \frac{R_n}{\sum_{n=1}^N R_n} V_s.$$

Symbols

V_n	Voltage difference over resistor n	V
R_n	Resistance over resistor n	Ω
V_s	Source voltage	V

3.2.5 Resistors in Parallel

$$R_{EQ} = \frac{1}{\sum_{n=1}^N \frac{1}{R_n}}.$$

Symbols

R_{eq}	Equivalent resistance	Ω
R_n	Resistance of resistor n	Ω
N	Amount of resistors in parallel	—

3.2.6 Current Divider Rule

$$I_n = \frac{\frac{1}{R_n}}{\sum_{n=1}^N \frac{1}{R_n}} I_s.$$

Symbols

I_n	Current difference over resistor n	A
R_n	Resistance over resistor n	Ω
I_s	Source current	A

Lecture 3: Principles of Circuit Analysis

2. Februar 2026

Lecture 4: Capacitors & Inductors

9. Februar 2026

5.1 Capacitors**5.1.1 Capacitance**

$$C = \frac{q}{V}.$$

Symbols

C	Capacitance	F
q	Charge	C
V	Voltage	V

5.1.2 Capacitance of a Parallel-Plate Capacitor

$$C = \epsilon_0 \frac{A}{d}.$$

Symbols

C	Capacitance	F
ϵ_0	Permittivity constant	$8,85 \cdot 10^{-12} \text{ C}^2/\text{Nm}^2$
A	Cross-sectional area	m^2
d	Separation distance	m

5.1.3 I-V Relation for an Ideal Capacitor

$$I_C(t) = C \frac{dV_C(t)}{dt} \iff V_C(t) = \frac{1}{C} \int_{-\infty}^t I_C(t') dt'.$$

Symbols

I_C	Current through capacitor	A
V_C	Voltage over capacitor	V
C	Capacitance	F
t	Time	s

5.1.4 Capacitance of a Cylindrical Capacitor

$$C = 2\pi\epsilon_0 \frac{L}{\ln \frac{b}{a}}.$$

Symbols

C	Capacitance	F
ϵ_0	Permittivity constant	$8,85 \cdot 10^{-12} \text{ C}^2/\text{Nm}^2$
L	Length	m
a	Radius of the inner post	m
b	Radius of the outer post	m

5.1.5 Capacitance of a Cylindrical Capacitor

$$C = 4\pi\epsilon_0 \frac{ab}{b-a}.$$

Symbols

C	Capacitance	F
ϵ_0	Permittivity constant	$8,85 \cdot 10^{-12} \text{ C}^2/\text{Nm}^2$
a	Radius of the inner sphere	m
b	Radius of the outer sphere	m

5.1.6 Parallel Capacitance

$$C_{\text{eq}} = \sum_{i=1}^n C_i.$$

Symbols

n	Number of capacitors	—
C_{eq}	Equivalent capacitance	F
C_i	Capacitance of capacitor i	F

5.1.7 Capacitors in Series

$$\frac{1}{C_{\text{eq}}} = \sum_{i=1}^n \frac{1}{C_i}.$$

Symbols

n	Number of capacitors	—
C_{eq}	Equivalent capacitance	F
C_i	Capacitance of capacitor i	F

5.1.8 Potential Energy in a Capacitor

$$U_E = \frac{1}{2} CV^2.$$

Symbols

U_E	Electric potential energy	J
C	Capacitance	F
V	Voltage	V

5.2 Inductors**5.2.1 Inductance**

$$L = n \frac{\Phi_B}{I}.$$

Symbols

L	Inductance	H
n	Number of windings	—
Φ_B	Magnetic flux	T
I	Current	A

5.2.2 I-V Relation for an Ideal Inductor

$$V_L(t) = L \frac{dI_L(t)}{dt} \iff I_L(t) = \frac{1}{L} \int_{t_0}^t V_L(t') dt' + I_0.$$

Symbols

I_L	Current through inductor	A
V_L	Voltage over inductor	V
L	Inductance	H
t	Time	s

5.2.3 Magnetic Field of an Inductor

$$B = \mu_0 In.$$

Symbols

B	Magnetic field vector	T
-----	-----------------------	---

μ_0	Magnetic vacuum permeability	$1,26 \cdot 10^{-6} \text{ N/A}^2$
I	Current	A
n	Number of windings	—

5.2.4 Inductors in Series

$$L_{\text{eq}} = \sum_{i=1}^n L_i.$$

Symbols

n	Number of inductors	—
L_{eq}	Equivalent Inductance	H
L_i	Inductance of inductor i	H

5.2.5 Parallel Inductors

$$\frac{1}{L_{\text{eq}}} = \sum_{i=1}^n \frac{1}{L_i}.$$

Symbols

n	Number of inductors	—
L_{eq}	Equivalent Inductance	H
L_i	Inductance of inductor i	H

5.2.6 Energy Stored in an Inductor

$$U_L = \frac{1}{2} L I^2.$$

Symbols

U_L	Potential energy stored in inductor	J
L	Inductance	H
I	Current	A

5.2.7 Energy Density in an Inductor

$$u_L = \frac{B^2}{2\mu_0}.$$

Symbols

u_L	Energy density of inductor	J/m^3
B	Magnetic field vector	T
μ_0	Magnetic vacuum permeability	$1,26 \cdot 10^{-6} \text{ N/A}^2$

5.3 RC and RL Circuits in the DC Regime

5.3.1 Charge in a (Charging) Capacitor as a Function of Time

$$q(t) = C V_s \left(1 - e^{-\frac{t}{RC}} \right).$$

Symbols

q	Charge	C
C	Capacitance	F
V_s	Source voltage	V
t	Time	s
R	Resistance	Ω
C	Capacitance	F

5.3.2 Current in a (Charging) Capacitor as a Function of Time

$$I_C(t) = \frac{V_s}{R} e^{-\frac{t}{RC}}.$$

Symbols

I_C	Current through capacitor	A
C	Capacitance	F
V_s	Source voltage	V
t	Time	s
R	Resistance	Ω
C	Capacitance	F

5.3.3 Capacitive Time Constant

$$\tau_C = RC.$$

Symbols

τ_C	Capacitive Time Constant	s
R	Resistance	Ω
C	Capacitance	F

5.3.4 Inductive Time Constant

$$\tau_L = \frac{L}{R}.$$

Symbols

τ_L	Inductive Time Constant	s
L	Inductance	H
R	Resistance	Ω

5.3.5 Current in an Inductor

$$I = \frac{V_s}{R} \left(1 - e^{-\frac{t}{\tau_L}}\right).$$

Symbols

I	Current	A
V_s	Source voltage	V
R	Resistance	Ω

t	Time	s
τ_L	Inductive Time Constant	s

Lecture 5: AC Circuits

11. Februar 2026

6.1 AC Circuits

6.1.1 Periodic Signals

$$x(t) = x(t + nT), \quad n = 1, 2, 3, \dots$$

Symbols

$x(t)$	Periodic signal x of t	—
t	Time	s
T	Period	s

6.1.2 Average Value of a Signal Waveform

$$\langle x(t) \rangle = \frac{1}{T} \int_0^T x(t') \, dt'.$$

Symbols

$x(t)$	Periodic signal x of t	—
$\langle x(t) \rangle$	Average value of $x(t)$	—
T	Period	s

6.1.3 RMS value

$$x_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T x^2(t') \, dt'}.$$

Symbols

x_{rms}	RMS of $x(t)$	—
$x(t)$	Periodic signal x of t	—
T	Period	s

6.2 Impedance

6.2.1 Impedance of a Resistor

$$Z(j\omega) = \frac{V(j\omega)}{I(j\omega)} = R.$$

Symbols

Z	Impedance	Ω
j	Imaginary unit	—
ω	Frequency	Hz
V	Voltage	V
I	Current	A

R	Resistance	Ω
-----	------------	----------

6.2.2 Impedance of an Inductor

$$Z(j\omega) = \frac{V(j\omega)}{I(j\omega)} = \omega L \angle \frac{\pi}{2}.$$

Symbols

Z	Impedance	Ω
j	Imaginary unit	—
ω	Frequency	Hz
V	Voltage	V
I	Current	A
L	Inductance	H

6.2.3 Impedance of a Capacitor

$$Z(j\omega) = \frac{V(j\omega)}{I(j\omega)} = \frac{1}{\omega C} \angle -\frac{\pi}{2} = \frac{1}{j\omega C}.$$

Symbols

Z	Impedance	Ω
j	Imaginary unit	—
ω	Frequency	Hz
V	Voltage	V
I	Current	A
C	Capacitance	F

6.2.4 Total Impedance

$$Z(j\omega) = R(j\omega) + jX(j\omega).$$

Symbols

Z	Impedance	Ω
j	Imaginary unit	—
ω	Frequency	Hz
R	Resistance	Ω
X	Reactance	Ω

6.2.5 Admittance

$$Y = \frac{1}{Z}.$$

Symbols

Y	Admittance	S
Z	Impedance	Ω

6.3 Transient Analysis

6.3.1 1st Order Differential Equation for RC Circuits

$$\tau \frac{dx(t)}{dt} + x(t) = K_s f(t).$$

Symbols

τ	$\tau = a_1 / a_0$ is a time constant	—
x	Voltage or current	—
t	Time	s
K_s	$K_s = b_0 / a_0$ is DC gain	—

6.3.2 2nd Order Differential Equation for RLC Circuits

$$\frac{1}{\omega_n^2} \frac{dx^2(t)}{dt^2} + \frac{2\zeta}{\omega_n} \frac{dx(t)}{dt} = K_s f(t).$$

Symbols

ω_n	$\omega_n = \sqrt{a_0 / a_2}$ is the natural frequency	Hz
ζ	$\zeta = a_1 / 2\sqrt{1/(a_0 a_2)}$ is the damping ratio	—
x	Voltage or current	—
t	Time	s
K_s	$K_s = b_0 / a_0$ is DC gain	—

Lecture 6: Frequency Analysis

16. Februar 2026

7.1 Frequency Response Function

$$H_V(j\omega) = \frac{V_L(j\omega)}{V_S(j\omega)}.$$

Symbols

H_V	Frequency response function	—
j	Imaginary unit	—
ω	Frequency	Hz
V_L	Output voltage	V
V_S	Source voltage	V

7.1.1 Phase Shift due to the Frequency Response Function

$$V_L e^{j\phi_L} = |H_V| V_S e^{j(\angle H_V + \phi_S)}.$$

Symbols

V_L	Output voltage	V
j	Imaginary unit	—
ϕ_L	Output phase	—
ϕ_L	Output phase, $\phi_L = \angle H_V + \phi_S$	—
H_V	Frequency response function	—

V_s	Source voltage	V
ϕ_s	Source phase	—

7.2 Passive Filters

7.2.1 Cutoff Frequency

$$\omega_0 = \frac{1}{RC}.$$

Symbols

ω_0	Cutoff angular frequency	rad/s
R	Resistance	Ω
C	Capacitance	F

7.2.2 Low-Pass Filter

Frequency Response

$$H_V(j\omega) = \frac{1}{1 + j\omega CR} = |H_V(j\omega)| e^{j\angle H_V(j\omega)}.$$

Symbols

H_V	Frequency response function	—
j	Imaginary unit	—
ω	Frequency	Hz
C	Capacitance	F
R	Resistance	Ω

Magnitude of the Frequency Response

$$|H_V(j\omega)| = \frac{1}{\sqrt{1 + (\omega CR)^2}} = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}.$$

Symbols

H_V	Frequency response function	—
j	Imaginary unit	—
ω	Angular frequency	rad/s
C	Capacitance	F
R	Resistance	Ω
ω_0	Cutoff angular frequency	rad/s

Phase of the Frequency Response

$$\angle H_V(j\omega) = -\tan^{-1}(\omega CR) = -\tan^{-1} \frac{\omega}{\omega_0}.$$

Symbols

H_V	Frequency response function	—
-------	-----------------------------	---

j	Imaginary unit	—
ω	Angular frequency	rad/s
C	Capacitance	F
R	Resistance	Ω
ω_0	Cutoff angular frequency	rad/s

7.2.3 High-Pass Filter

Frequency Response

$$H_V(j\omega) = \frac{j\omega CR}{1 + j\omega CR} = |H_V(j\omega)| e^{j\angle H_V(j\omega)}.$$

Symbols

H_V	Frequency response function	—
j	Imaginary unit	—
ω	Frequency	Hz
C	Capacitance	F
R	Resistance	Ω

Magnitude of the Frequency Response

$$|H_V(j\omega)| = \frac{\omega CR}{\sqrt{1 + (\omega CR)^2}}.$$

Symbols

H_V	Frequency response function	—
j	Imaginary unit	—
ω	Angular frequency	rad/s
C	Capacitance	F
R	Resistance	Ω

Phase of the Frequency Response

$$\angle H_V(j\omega) = 90^\circ - \tan^{-1}(\omega CR).$$

Symbols

H_V	Frequency response function	—
j	Imaginary unit	—
ω	Angular frequency	rad/s
C	Capacitance	F
R	Resistance	Ω

7.2.4 Bandpass Filter

Frequency Response

$$H_V(j\omega) = \frac{j\omega CR}{1 + j\omega CR + (j\omega)^2 LC} = \frac{jCR\omega}{\left(j\frac{\omega}{\omega_1} + 1\right)\left(j\frac{\omega}{\omega_2} + 1\right)}.$$

Symbols

H_V	Frequency response function	—
j	Imaginary unit	—
ω	Angular frequency	rad/s
C	Capacitance	F
R	Resistance	Ω
L	Inductance	H
ω_1	Lower angular cutoff frequency	rad/s
ω_2	Upper angular cutoff frequency	rad/s

Magnitude of the Frequency Response

$$|H_V(j\omega)| = \frac{CR\omega}{\sqrt{\left(1 + \left(\frac{\omega}{\omega_1}\right)^2\right)\left(1 + \left(\frac{\omega}{\omega_2}\right)^2\right)}}.$$

Symbols

H_V	Frequency response function	—
j	Imaginary unit	—
ω	Angular frequency	rad/s
C	Capacitance	F
R	Resistance	Ω
ω_1	Lower angular cutoff frequency	rad/s
ω_2	Upper angular cutoff frequency	rad/s

Phase of the Frequency Response

$$\angle H_V(j\omega) = \frac{\pi}{2} - \tan^{-1} \frac{\omega}{\omega_1} - \tan^{-1} \frac{\omega}{\omega_2}.$$

Symbols

H_V	Frequency response function	—
j	Imaginary unit	—
ω	Angular frequency	rad/s
ω_1	Lower angular cutoff frequency	rad/s
ω_2	Upper angular cutoff frequency	rad/s

Natural or Resonant Frequency

$$\omega_n = \sqrt{\frac{1}{LC}}.$$

Symbols

ω_0	Resonant or natural angular frequency	rad/s
L	Inductance	H
C	Capacitance	F

Quality Factor

$$Q = \frac{2}{\zeta} = \frac{1}{\omega_n CR} = \frac{1}{R} \sqrt{\frac{L}{C}}.$$

Symbols

Q	Quality	—
ζ	Damping Ratio	—
ω_0	Resonant or natural angular frequency	rad/s
C	Capacitance	F
R	Resistance	Ω
L	Inductance	H

Damping Ratio

$$\zeta = \frac{1}{2Q} = \frac{R}{2} \sqrt{\frac{C}{L}}.$$

Symbols

ζ	Damping Ratio	—
Q	Quality	—
R	Resistance	Ω
C	Capacitance	F
L	Inductance	H

Bandwidth

$$B = \frac{\omega_n}{Q}.$$

Symbols

B	Bandwidth	rad/s
ω_0	Resonant or natural angular frequency	rad/s
Q	Quality	—

Lecture 7: Diodes

25. Februar 2026

8.1 Diodes**8.1.1 Net Diode Current in forward bias mode**

$$I_D = I_0 \left(e^{V_D \frac{q}{kT}} - 1 \right).$$

Symbols

I_D	Current across diode	A
I_0	Leakage current at low voltage	A
q	Charge of an electron	$1,6 \cdot 10^{-19} \text{ C}$
V_D	Voltage across diode	V
k	Boltzman's constant	$1,3 \cdot 10^{-23} \frac{\text{J}}{\text{K}}$
T	Temperature	K